Midterm 2 review

Set theory

- *Set builder notation*
- *Subsets*
  - *Proving A = B by showing each are subsets of the other*
- *Set operations (incl. Power set and cartesian products)*
  - *Union, intersection, complement, difference, symmetric difference*
- *Set laws*
  - *Helps to remember the connection to logic*

Relations

- *Definition of an n-ary relation*
  - *Subset of n-ary cartesian product*
- *Representations of relations*
  - *Set of tuples, matrix, graph*
- *Properties of relations*
  - *Reflexivity, transitivity, symmetricity, anti-symmetricity*
  - *+ How to spot these from a graph or matrix*
  - *+ How to prove from a description of the relation*
- *Types of relations*
  - *Equivalences, partial orders, total orders*
  - *+ How to prove*

Functions

- *Definition*
  - *Binary relation f from A to B where each element a of A appears in exactly one pair (a,f(a))*
- *Representation of functions*
- *Terminology*
  - *Domain, codomain, range, image, preimage, restriction, extension*
- *Properties of functions*
  - *One-to-one/injective, onto/surjective, bijective*
- *Composition of functions*
  - *Remember the order!* $(f \circ g)(x) = f(g(x))$*, i.e. right-to-left, so g is* **first**
- *Inverse functions*

Cardinality and counting

- *(relative) Cardinality of a set*
  - $|A| = |B|$ *if bijection from A to B*
  - $|A| \leq |B|$ *if injection from A to B*
- *Countable sets*
  - *Definition – injection from A to N*
  - *How to prove – demonstrate an injection (i.e. "count" the elements)*
  - *Examples: N,Z,Q*
- *Uncountable sets*
  - *Definition – no injection from A to N*
  - *How to prove – diagonalization*
  - *Examples: R, P(N)*

Induction

- *Principle of mathematical induction*
  - $(P(1) \wedge \forall k\, (P(k) \rightarrow P(k+1))) \rightarrow \forall n \in \mathbb{Z}^+\, P(n)$
- *Proof by induction*
  - *Base case: show P(1) (or P(0))*
  - *Inductive step: assume P(k) and show P(k+1)*
- *Strong induction*
  - $(P(1) \wedge \forall k\, (P(1) \wedge P(2) \wedge \ldots \wedge P(k) \rightarrow P(k+1))) \rightarrow \forall n \in \mathbb{Z}^+\, P(n)$
- *Well-orderings*
- *Recursive functions*
- *Inductive definitions*
  - *Base case: a single dog is a pack of dogs*
  - *Inductive case: two packs of dogs is a pack of dogs*
- *Structural induction*