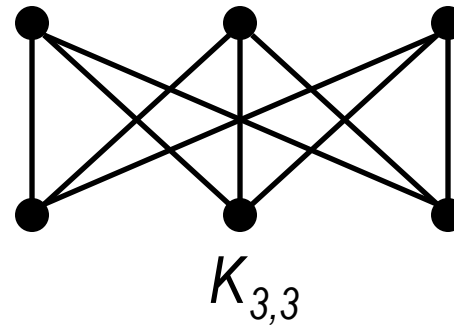
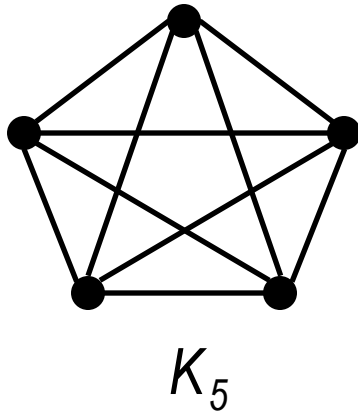


Review

Non-Planar Graphs

- We ended off showing that Euler's formula $v - e + r = 2$ can be used to show that certain graphs can NOT be planar
- The graph K_5 is not planar.
- The graph $K_{3,3}$ is also non-planar



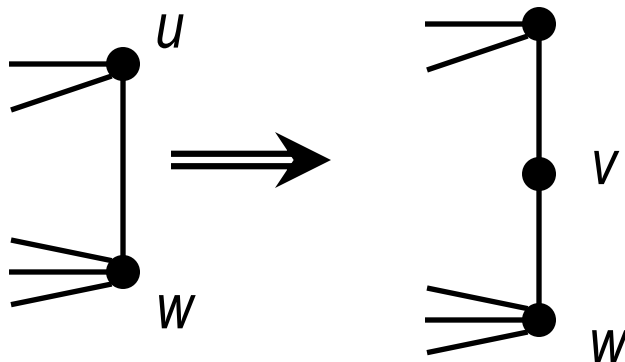
- How do we know when a graph is non-planar in general?

Kuratowski's Theorem

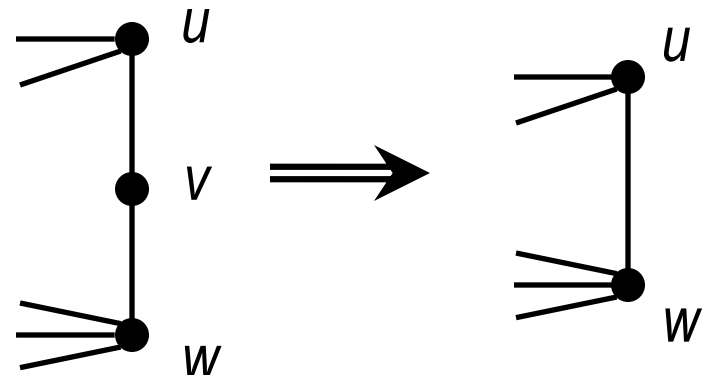
● Obviously, if a graph has a subgraph which is non-planar, it is also non-planar

● **Kuratowski's Theorem.**

A graph is nonplanar if and only if it contains a subgraph that can be obtained from either K_5 or $K_{3,3}$ by a sequence of **subdivisions** and **exclusions**

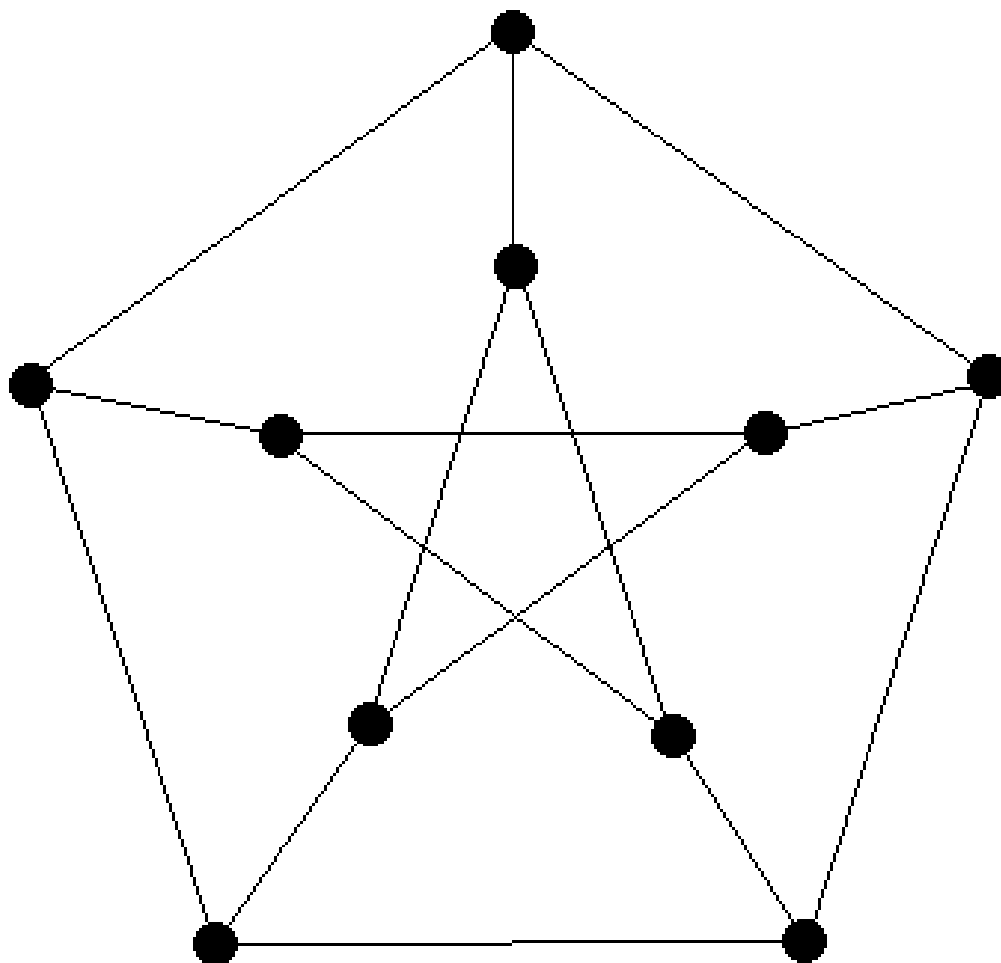


subdivision



exclusion

Petersen Graph



Review - Integers

- *Definition (+notation) of integers, positive integers, natural numbers*
- *Definition & properties of the divisibility relation*
 - $a \mid b$ if and only if $b = ad$
- *Division algorithm (Euclidean division)*
 - $a = bq + r$ where $0 \leq r < q$
- *Base b expansion*
- *Definition of prime/composite/relatively prime numbers*
- *GCD and LCM*
 - Note: greatest/least means *in the divisibility relation*, NOT the integer order $\leq$
- *Euclidean algorithm*
 - Both parts --- finding the GCD and using the result to write $\gcd(a,b) = ua + vb$
- *Fundamental theorem of arithmetic*

Modular arithmetic

- *Definition (+ equivalent characterizations) of congruence modulo m*
 - $a \equiv b \pmod{m}$ if $m \mid (a - b)$
 - ... if a and b have the same remainder when divided by m
 - ... if $a = km + b$ for some k
- *Definitions of residues and integers modulo m*
- *Modular arithmetic*
 - Addition, subtraction, and multiplication modulo m
 - Inverses modulo m
 - When does a have an inverse modulo m ? (when a, m are relatively prime)
- *Chinese remainder theorem*
 - + How to use it to solve systems of linear congruences

Graph theory

- *Definitions of directed, undirected, and multi graphs*
- *Handshaking lemma*
- *Definitions of walks, paths, cycles/loops, connected components*
- *Graph representations*
 - *Adjacency list, Adjacency matrix, Incidence matrix*
- *Graph isomorphism*
 - *How to prove non-isomorphic via graph invariants*
- *Subgraphs*
- *Bipartite graphs*
 - *Characterization via non-existence of odd-length cycles*
- *Trees, rooted trees*
 - *Properties: connected, no cycles, $n-1$ edges*
 - *How to do inductive proofs*
- *Planar graphs*
 - *Euler's formula $v - e + r = 2$*

Thanks everyone!