

Last Name _____ First Name and Initials _____

Student No. _____

NO AIDS allowed. Answer ALL questions on the test paper. Use backs of sheets for scratch work.

Total Marks: 100

1. Let $f : \mathbb{Z} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2$. What is the domain of f ? What is the codomain? What is the range? [10]
2. Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions such that $g \circ f$ is an injection. Do f and g both have to be injections? Prove your answer or give a counterexample. [10]
3. Relation R is given by matrix [16]

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}.$$

Is R an order? If yes, what its minimal, maximal, least, and greatest elements are?

4. Make a list of pairs, construct the matrix, and draw the graph of the relation R from the set $A = \{0, 1, 2, 3, 4\}$ to the set $B = \{0, 1, 2, 3\}$ such that $(a, b) \in R$ if and only if $a + b = 4$. [12]
5. Prove that the rational numbers $\mathbb{Q}$ are countable.. [10]
6. The Fibonacci numbers are defined inductively as

$$f(1) = 1 \quad f(2) = 1 \quad f(n) = f(n-1) + f(n-2).$$

Prove by strong induction that for every $n \geq 3$,

$$f(n) = 1 + \sum_{i=0}^{n-2} f(i)$$

[16]

7. Give a definition of a transitive relation. Give an example of a relation which is NOT transitive. [10]
8. Let A , B , and C be sets. Show that [16]

$$(A - B) - C \subseteq A - C.$$

Draw Venn diagrams for both expressions.