

Last Name _____ First Name and Initials _____

Student No. _____

NO AIDS allowed. Answer ALL questions on the test paper. Use backs of sheets for scratch work.

Total Marks: 100

1. Can a (partial) order have two greatest elements? Two least elements? Explain. [10]
2. Can the composition of two injective functions be a non-injective function? Explain. [12]
3. Show that the relation on $A = \{a, b, c, d, e, f\}$ given by the following matrix (the rows and columns are labeled by a, b, c, d, e, f , in this order, is an equivalence relation (skip a proof of transitivity) and find its equivalence classes. [18]

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

4. Define a function $f : \mathbb{R} \rightarrow \mathbb{R}$ by the formula $f(x) = 3x - 5$. Is this function surjective? Prove your answer. [10]
5. Give a definition of a countable set and provide 3 examples of uncountable sets. [12]
6. What is the difference in the Induction Hypothesis in regular and strong induction? [12]
7. Prove that for any sets A , B , and C [12]

$$A \cup \overline{B} \subseteq (A \cap B) \cup \overline{(B \cap C)}.$$

Draw Venn diagrams of both sides of the inequality.

8. State the well-ordering principle. Is it true for the set of all positive rational numbers? Explain. [14]