

Tutorial problems — MACM101 (Summer 2026), Week 11

1. Which positive integers less than 30 are relatively prime with 30?
2. For positive integers a, b and $d = \gcd(a, b)$, prove that

$$\gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1.$$

3. Let n be a positive integer. Prove that $\gcd(n, n+2)$ equals 1 or 2.
4. Find the greatest common divisor of 168 and 456, and also numbers u, v such that $\gcd(168, 456) = 168u + 456v$.
5. Find the greatest common divisor and the least common multiple of 630 and 40452.
6. Find the prime factorization of
(a) 148500; (b) 7114800; (c) 7882875.
7. How many positive divisors are there for

$$n = 2^{14}3^95^87^{10}11^313^537^{10}.$$

8. A perfect square is a number n such that $n = k^2$ for some integer k . Determine the smallest perfect square that is divisible by 7!.
9. Show that if $2^n - 1$ is prime then n is prime.
(*Hint: use the identity $a^x - 1 = (a - 1)(a^{x-1} + a^{x-2} + \cdots + a + 1)$.*)