Tutorial problems — MACM101 (Summer 2026), Week 12

1. Determine whether each of the following pairs of integers are congruent modulo 9.

$$(i)\ 76, 243 \qquad (ii)\ -137, 700 \qquad (iii)\ -56, -1199$$

2. For each of the following determine the values of the integers $n > 1$ for which the given congruence is true

$$(a)\ 28 \equiv 6 \pmod{n} \qquad (b)\ 68 \equiv 37 \pmod{n}$$

3. Let $n \in \mathbb{Z}$. Prove that $n(n+1)(n+2)$ is divisible by 3.

4. Let $m, n \in \mathbb{N}$ with $gcd(m, n) = 1$ and let $a, b \in \mathbb{Z}$. Prove that $a \equiv b \pmod{m}$ and $a \equiv b \pmod{n}$ if and only if $a \equiv b \pmod{mn}$.

5. If $n \in \mathbb{N}$ and $n > 2$ prove that

$$\sum_{i=1}^{n-1} i = \begin{cases} 0 \pmod{n}, & n \text{ odd} \\ \frac{n}{2} \pmod{n}, & n \text{ even} \end{cases}$$

6. How many proper divisors of 0 are there in $\mathbb{Z}_{17}$, in $\mathbb{Z}_{25}$.

7. Find the inverse of $5 \pmod{13}$. Find the inverse of $22 \pmod{27}$.

8. Solve the following system of linear congruences

$$\begin{aligned} x &\equiv 2 \pmod{11} \\ x &\equiv 1 \pmod{7}. \end{aligned}$$

9. For $a, b, n \in \mathbb{Z}$ and $n > 1$, prove that $a \equiv b \pmod{n}$ implies $gcd(a, n) = gcd(b, n)$.