Tutorial problems — MACM101 (Summer 2026), Week 3

1. Show that the following compound statements are logically equivalent:
   - $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$;
   - $\neg p \to (q \to r)$ and $q \to (p \vee r)$.
2. Is $(p \vee q) \to (q \to (p \wedge q))$ a contradiction?
3. Verify that
$$(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p) \Leftrightarrow (p \to q) \wedge (q \to r) \wedge (r \to p).$$
4. Negate the following statement and simplify the result $p \vee q \vee (\neg p \wedge \neg q \wedge r)$.
5. Let "Nand" be the logic connective defined by $p \uparrow q \iff \neg(p \wedge q)$. Express $\neg, \vee, \wedge$ using only Nand. (difficult problem, not for everyone)
6. Verify that the Rule of Syllogism is a valid argument. (Use the corresponding tautology.)
7. Verify that the following is a tautology by showing that it is impossible for the conclusion to have truth value 0 while the premises have truth value 1:
$$((p \to q) \wedge (r \to s) \wedge (p \vee r)) \to (q \vee s).$$
8. For each of the following pairs of statements use Modus Ponens or Modus Tollens to make a valid argument.
   - "If Janice has trouble starting her car, then her daughter Angela will check Janice's spark plugs.
     Janice had trouble starting her car."
   - "If Brady solved the first problem correctly, then the answer he obtained is 137.
     Brady's answer to the first problem is not 137."
9. Give the reasons for each step needed to show that the following argument is valid
   Premises: $p, p \to q, s \vee r, r \to \neg q$
   Conclusion: s.

Steps	Reasons
1. p	
2. $p \to q$	
3. q	
4. $r \to \neg q$	
5. $q \to \neg r$	
6. $\neg r$	
7. $s \vee r$	
8. s	

10. Solve problems 5,6 for the previous Tutorial using rules of inference. (difficult problem, not for everyone)