Tutorial problems — MACM101 (Summer 2026), Week 4

1. Consider the universe of all polygons with three or four sides, and define the following predicates for this universe:

   $A(x)$: all interior angles of x are equal;
   $E(x)$: x is a equilateral triangle;
   $H(x)$: all sides of x are equal;
   $P(x)$: x has an interior angle that exceeds $180°$;
   $Q(x)$: x is a quadrilateral;
   $R(x)$: x is a rectangle;
   $S(x)$: x is a square;
   $T(x)$: x is a triangle.

   Translate each of the following statements into an English sentence, and determine whether the statement is true or false.

   (a) $\forall x\ (Q(x) \oplus T(x))$;
   (b) $\exists x\ (T(x) \wedge P(x))$;
   (c) $\exists x\ (Q(x) \wedge \neg R(x))$;
   (d) $\forall x\ (H(x) \rightarrow E(x))$;
   (e) $\forall x\ (S(x) \leftrightarrow (A(x) \wedge H(x)))$.

2. Let $P(x, y), Q(x, y)$ denote the following predicates

   $$P(x, y): \quad x^2 \geq y \qquad\qquad Q(x, y): \quad x + 2 < y.$$

   If the universe for each of x, y consists of all real numbers, determine the truth value for each of the following statements

   (a) $P(-3, 8) \wedge Q(1, 3)$;
   (b) $P(\frac{1}{2}, \frac{1}{3}) \vee \neg Q(-2, -3)$;
   (c) $P(1, 2) \leftrightarrow \neg Q(1, 2)$.

3. Let $P(x), Q(x)$, and $R(x)$ denote the following predicates

   $$P(x): \quad x^2 - 8x + 15 = 0 \qquad Q(x): \quad x \text{ is odd} \qquad R(x): \quad x > 0.$$

   If the universe for x consists of all integers, determine the truth value for each of the following statements. If a statement is false, give a counterexample.

   (a) $\forall x\ (P(x) \rightarrow Q(x))$;
   (b) $\exists x\ (P(x) \rightarrow Q(x))$;

(c) $\exists x\ (R(x) \to P(x))$;
(d) $\forall x\ ((P(x) \vee Q(x)) \to R(x))$.

4. Write the negation of each of the following statements as an English sentence — without symbolic notation. (Here the universe consists of all the students at the university where Professor Lenhart teaches.)

   (a) Every student in Professor Lenhart's C++ class is majoring in computer science or mathematics.
   (b) At least one student in Professor Lenhart's C++ class is a history major.

5. Negate and simplify each of the following.

   (a) $\forall x\ (P(x) \to Q(x))$;
   (b) $\exists x\ ((P(x) \vee Q(x)) \to R(x))$.

6. Determine the truth value of each statement in the case when the universe comprises all nonzero integers, and in the case when the universe consists of all nonzero real numbers.

   (a) $\exists x \forall y\ (xy = 1)$;
   (b) $\forall x \exists y\ ((2x + y = 5) \wedge (x - 3y = -8))$.

7. Let the universe for the variables in the following statements consists of all real numbers. In each case negate and simplify the given statement.

   (a) $\forall x \forall y\ ((x < y) \to \exists z\ (x < z < y))$;
   (b) $(\forall x \forall y\ ((x > 0) \wedge (y > 0))) \to (\exists z\ (xz > y))$.