

1. Negate and simplify each of the following.
  - (a)  $\forall x (P(x) \rightarrow Q(x))$ ;
  - (b)  $\exists x ((P(x) \vee Q(x)) \rightarrow R(x))$ .
  
2. Let the universe for the variables in the following statements consists of all real numbers. In each case negate and simplify the given statement.
  - (a)  $\forall x \forall y ((x < y) \rightarrow \exists z (x < z < y))$ ;
  - (b)  $(\forall x \forall y ((x > 0) \wedge (y > 0))) \rightarrow (\exists z (xz > y))$ .
  
3. Determine which of the following arguments are valid and which are invalid. Provide an explanation for each answer. (Let the universe consist of all people presently residing in Canada.)
  - (a) All mail carriers carry a can of mace.  
Mrs. Bacon is a mail carrier.  
Therefore Mrs. Bacon carries a can of mace.
  - (b) All law-abiding citizens pay their taxes.  
Mr. Pelosi pays his taxes.  
Therefore Mr. Pelosi is a law-abiding citizen.
  - (c) All people who are concerned about the environment recycle their plastic containers.  
Margarite is not concerned about the environment.  
Therefore Margarite does not recycle her plastic containers.

4. For a prescribed universe and any propositions  $P(x), Q(x)$ , prove that

$$\forall x (P(x) \wedge Q(x)) \iff (\forall x P(x)) \wedge (\forall x Q(x)).$$

5. Provide the reasons for the steps verifying the following argument. (Here  $a$  denotes a specific but arbitrary chosen element from the given universe.) (For Step 9 refer to Table 1 (Section 1.6) in the textbook.)  
Premises:  $\forall x (P(x) \rightarrow (Q(x) \wedge R(x)))$ ,  $\forall x (P(x) \wedge S(x))$ .  
Conclusion:  $\forall x (R(x) \wedge S(x))$ .

Steps	Reasons
1. $\forall x (P(x) \rightarrow (Q(x) \wedge R(x)))$	
2. $\forall x (P(x) \wedge S(x))$	
3. $P(a) \rightarrow (Q(a) \wedge R(a))$	
4. $P(a) \wedge S(a)$	
5. $P(a)$	
6. $Q(a) \wedge R(a)$	
7. $R(a)$	
8. $S(a)$	
9. $R(a) \wedge S(a)$	
10. $\forall x (R(x) \wedge S(x))$ .	

6. Give a direct proof that, for all integers  $k$  and  $\ell$ , if  $k, \ell$  are both even, then  $k + \ell$  is even.
7. Give a proof by contraposition that, for all integers  $k$  and  $\ell$ , if  $k\ell$  is odd, then  $k, \ell$  are both odd.