

Tutorial problems — MACM101 (Summer 2026), Week 6

1. Let $A = \{1, \{1\}, \{2\}\}$. Which of the following statements are true?

- | | |
|----------------------------|----------------------------|
| a) $1 \in A$ | b) $\{1\} \in A$ |
| c) $\{1\} \subseteq A$ | d) $\{\{1\}\} \subseteq A$ |
| e) $\{2\} \in A$ | f) $\{2\} \subseteq A$ |
| g) $\{\{2\}\} \subseteq A$ | |

2. For $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ let $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 2, 4, 8\}$, $C = \{1, 2, 3, 5, 7\}$, and $D = \{2, 4, 6, 8\}$. Determine each of the following:

- | | |
|------------------------------|--------------------------|
| a) $(A \cup B) \cap C$ | b) $A \cup (B \cap C)$ |
| c) $\overline{C \cup D}$ | d) $\overline{C \cap D}$ |
| e) $(A \cup B) - C$ | f) $A \cup (B - C)$ |
| g) $(B - C) - D$ | h) $B - (C - D)$ |
| i) $(A \cup B) - (C \cap D)$ | |

Draw Venn diagrams for each of the expressions.

3. Demonstrate each of the following using Venn diagrams. (Assume a universe U .)

- If $A \subseteq B$ and $C \subseteq D$, then $A \cap C \subseteq B \cap D$ and $A \cup C \subseteq B \cup D$.
- $A \subseteq B$ if and only if $A \cap \overline{B} = \emptyset$.
- $A \subseteq B$ if and only if $\overline{A} \cup B = U$.

4. Prove or disprove each of the following:

- For sets $A, B, C \subseteq U$, if $A \cup C = B \cup C$, then $A = B$.
- For sets $A, B, C \subseteq U$, if $A \cup C = B \cup C$ and $A \cap C = B \cap C$, then $A = B$.

5. Prove that $A - B = A \cap \overline{B}$.

6. Prove that $A \Delta B = (\overline{A} \cap B) \cup (A \cap \overline{B})$.

7. Investigate the truth or falsity of the following using 3 methods: Venn diagrams, laws of set theory, and proving that the left side is a subset of the right one and vice versa.

$$A - (B \cup C) = (A - B) \cap (A - C).$$