

Tutorial problems — MACM101 (Summer 2026), Week 8

- Determine whether or not the following relations are functions. If a relation is a function, find its range.
 - $\{(x, y) \mid x, y \in \mathbb{Z}, y = x^2 + 7\}$, a relation from \mathbb{Z} to \mathbb{Z} ;
 - $\{(x, y) \mid x, y \in \mathbb{R}, y^2 = x\}$, a relation from \mathbb{R} to \mathbb{R} .
- For each of the following functions, determine whether it is one-to-one and determine its range.
 - $f: \mathbb{Z} \rightarrow \mathbb{Z}, f(x) = 2x + 1$;
 - $f: \mathbb{Q} \rightarrow \mathbb{Q}, f(x) = 2x + 1$;
 - $f: \mathbb{Z} \rightarrow \mathbb{Z}, f(x) = x^3 - x$;
- Let $f: A \rightarrow B$ where $A = X \cup Y$ with $X \cap Y = \emptyset$. If $f|_X$ and $f|_Y$ are one-to-one, does it follow that f is one-to-one?
- For each of the following functions $f: \mathbb{Z} \rightarrow \mathbb{Z}$, determine whether the function is onto. If the function is not onto, determine the range of f .
 - $f(x) = 2x - 3$;
 - $f(x) = x^2 + x$.
- Let $f, g: \mathbb{R} \rightarrow \mathbb{R}$, where $g(x) = 1 - x + x^2$ and $f(x) = ax + b$. If $(g \circ f)(x) = 9x^2 - 9x + 3$, determine a, b .
- Let $g: \mathbb{N} \rightarrow \mathbb{N}$ be defined by $g(n) = 2n$. If $A = \{1, 2, 3, 4\}$ and $f: A \rightarrow \mathbb{N}$ is given by $f = \{(1, 2), (2, 3), (3, 5), (4, 7)\}$, find $g \circ f$.
- If $f \circ g$ is onto, does it follow that g is onto?
- For each of the following functions $f: \mathbb{R} \rightarrow \mathbb{R}$, determine whether f is invertible, and, if so, determine f^{-1} .
 - $f = \{(x, y) \mid 2x + 3y = 7\}$;
 - $f = \{(x, y) \mid y = x^4 + x\}$.