Tutorial problems — MACM101 (Summer 2026), Week 10

1. A jigsaw puzzle is put together by successively joining pieces that fit together into blocks. A move is made each time a piece is added to a block, or when two blocks are joined. Use strong induction to prove that no matter how the moves are carried out, exactly $n-1$ moves are required to assemble a puzzle with n pieces.

2. Give a recursive definition of $P_m(n)$, the product of the integer m and the non-negative integer n.

3. Let $F(n)$ be the nth Fibonacci number. Prove that

$$F(1) + F(2) + \cdots + F(n) = F(n+2) - 1.$$

4. A complete binary tree is a graph defined through the following recursive definition.
   Basis step: A single vertex is a complete binary tree.
   Inductive step: If T_1 and T_2 are disjoint complete binary trees with roots r_1, r_2, respectively, then the graph formed by starting with a root r, and adding an edge from r to each of the vertices r_1, r_2 is also a complete binary tree.
   Draw all complete binary trees that can be obtain by applying the inductive step at most 3 times (complete binary trees of level 3).
   Use structural induction to show that $n(T) \geq 2h(T) + 1$, where $n(T)$ denotes the number of vertices in T, and $h(T)$ denotes the height of T.

5. Determine the quotient q and the remainder r for each of the following, where a is the dividend and b is the divisor.
   (a) $a = 123$, $b = 17$; (b) $a = -115$, $b = 12$;
   (c) $a = 0$, $b = 42$; (d) $a = 434$, $b = 31$.

6. Write each of the following 10-base numbers in base 2 and 16
   (a) 137; (b) 6243; (c) 12345.

7. Convert each of the following binary numbers to base 10
   (a) 11001110; (b) 00110001.