Tutorial solutions — MACM101 (Summer 2026), Week 1

Throughout, T means true and F means false.

Problem 1 (Converse, contrapositive, and inverse)**.** State the converse, contrapositive, and inverse of each implication:

- If it snows today, I will ski tomorrow.
- I come to class whenever there is going to be a quiz.
- A positive integer is a prime only if it has no divisors other than 1 and itself.

Answer.

For the first implication, let

$$\underbrace{\text{it snows today}}_{S}, \qquad \underbrace{\text{I will ski tomorrow}}_{K}.$$

The original statement is $S \to K$: if it snows today, then I will ski tomorrow.

- Converse: $K \to S$. If I ski tomorrow, then it snowed today.
- Contrapositive: $\neg K \to \neg S$. If I do not ski tomorrow, then it does not snow today.
- Inverse: $\neg S \to \neg K$. If it does not snow today, then I will not ski tomorrow.

The statement equivalent to the original is its contrapositive: if I do not ski tomorrow, then it does not snow today. The converse is equivalent to the inverse.
For the second implication, let

$$\underbrace{\text{there is going to be a quiz}}_{Q}, \qquad \underbrace{\text{I come to class}}_{C}.$$

The original statement is $Q \to C$: if there is going to be a quiz, then I come to class.

- Converse: $C \to Q$. If I come to class, then there is going to be a quiz.
- Contrapositive: $\neg C \to \neg Q$. If I do not come to class, then there is not going to be a quiz.
- Inverse: $\neg Q \to \neg C$. If there is not going to be a quiz, then I do not come to class.

The statement equivalent to the original is its contrapositive: if I do not come to class, then there is not going to be a quiz. The converse is equivalent to the inverse.
For the third implication, let

$$\underbrace{\text{the positive integer is prime}}_{P}, \qquad \underbrace{\text{it has no divisors other than 1 and itself}}_{D}.$$

The original statement is $P \to D$: if a positive integer is prime, then it has no divisors other than 1 and itself.

- Converse: $D \to P$. If a positive integer has no divisors other than 1 and itself, then it is prime.

- Contrapositive: $\neg D \to \neg P$. If a positive integer has a divisor other than 1 and itself, then it is not prime.
- Inverse: $\neg P \to \neg D$. If a positive integer is not prime, then it has a divisor other than 1 and itself.

The statement equivalent to the original is its contrapositive: if a positive integer has a divisor other than 1 and itself, then it is not prime. The converse is equivalent to the inverse.

Problem 2 (Access system specification)**.** Express the following system specification using propositional logic, and construct the truth table for the compound proposition: "Access is granted whenever the user has paid the subscription fee and enters a valid password."

Answer.

Let

$$\underbrace{\text{access is granted}}_{A},$$
$$\underbrace{\text{the user has paid the subscription fee}}_{P},$$
$$\underbrace{\text{the user enters a valid password}}_{V}.$$

The specification is

$$(P \wedge V) \to A.$$

P	V	A	$P \wedge V$	$(P \wedge V) \to A$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

Problem 3 (Consistency of system specifications)**.** Are the following system specifications consistent? "The system is in multiuser state if and only if it is operating normally. If the system is operating normally, the kernel is functioning. The kernel is not functioning or the system is in interrupt mode. If the system is not in multiuser state, then it is in interrupt mode. The system is not in interrupt mode."

Answer.

Let

$$\underbrace{\text{the system is in multiuser state}}_{M}, \quad \underbrace{\text{the system is operating normally}}_{N},$$
$$\underbrace{\text{the kernel is functioning}}_{K}, \quad \underbrace{\text{the system is in interrupt mode}}_{I}.$$

The specifications are

$$M \leftrightarrow N, \qquad N \to K, \qquad \neg K \lor I, \qquad \neg M \to I, \qquad \neg I.$$

They are inconsistent. Indeed, from $\neg I$ and $\neg K \lor I$ we get $\neg K$. From $N \to K$ and $\neg K$ we get $\neg N$. From $M \leftrightarrow N$ we get $\neg M$. From $\neg M \to I$ we get I, contradicting $\neg I$.

Problem 4 (Truth tables for formulas)**.** Construct the truth tables for the following formulas:

- $(p \to q) \to (q \to p)$.
- $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$.
- $(p \to q) \land (\neg p \to q)$.

Answer.

For $(p \to q) \to (q \to p)$:

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

For $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$:

p	q	$p \leftrightarrow q$	$\neg p \leftrightarrow q$	$(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	T	F	T

For $(p \to q) \land (\neg p \to q)$:

p	q	$p \to q$	$\neg p \to q$	$(p \to q) \land (\neg p \to q)$
T	T	T	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

Problem 5 (Knights and knaves)**.** There are two tribes living on the island of Knights and Knaves: knights and knaves. Knights always tell the truth and knaves always lie. You encounter two people A and B. What are A and B if A says, "B is a knight", and B says, "The two of us are opposite types"?

Answer.

Let

$$\underbrace{A \text{ is a knight}}_{A}, \qquad \underbrace{B \text{ is a knight}}_{B}.$$

The statements give

$$A \leftrightarrow B, \qquad B \leftrightarrow (A \oplus B).$$

The only solution is $A = F$ and $B = F$. Hence both are knaves.

Problem 6 (Knights and knaves again)**.** There are two tribes living on the island of Knights and Knaves: knights and knaves. Knights always tell the truth and knaves always lie. You encounter two people A and B. What are A and B if A says, "The two of us are both knights", and B says, "A is a knave"?

Answer.

Let

$$\underbrace{A \text{ is a knight}}_{A}, \qquad \underbrace{B \text{ is a knight}}_{B}.$$

The statements give

$$A \leftrightarrow (A \wedge B), \qquad B \leftrightarrow \neg A.$$

The only solution is $A = F$ and $B = T$. Hence A is a knave and B is a knight.

Problem 7 (Absorption law)**.** Use truth tables to verify the absorption law.

Answer.

The absorption laws are

$$p \vee (p \wedge q) \equiv p, \qquad p \wedge (p \vee q) \equiv p.$$

p	q	$p \wedge q$	$p \vee (p \wedge q)$	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	F	F	T	F
F	F	F	F	F	F

The fourth and sixth columns agree with p.

Problem 8 (Hypothetical syllogism)**.** Show that $((p \to q) \wedge (q \to r)) \to (p \to r)$ is a tautology.

Answer (Answer, algebraically.).

$$\begin{aligned}((p \to q) \land (q \to r)) \to (p \to r) &\equiv \neg((\neg p \lor q) \land (\neg q \lor r)) \lor (\neg p \lor r) \\ &\equiv (p \land \neg q) \lor (q \land \neg r) \lor \neg p \lor r \\ &\equiv \big(\neg p \lor (p \land \neg q)\big) \lor \big(r \lor (q \land \neg r)\big) \\ &\equiv (\neg p \lor \neg q) \lor (r \lor q) \\ &\equiv T.\end{aligned}$$

Answer (Answer, by truth table.).

p	q	r	$p \to q$	$q \to r$	$p \to r$	$((p \to q) \land (q \to r)) \to (p \to r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	F	T	T	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

Hence the formula is a tautology.