1 Evaluate Infinite Loop

Provide the derivation for two steps of evaluation of the following program:

$$(\text{fix} \ (\lambda(f : () \rightarrow A). \ \lambda(x : ()).f \ x)) ()$$

2 Write Recursive Code

Recall the shorthand of $\mathbb{N} = \mu \text{nat.}() + \text{nat}$ and $\mathbb{L} = \mu \text{list.}() + (\mathbb{N} \times \text{list})$.

Write a function that returns the “last” element of the list. In other words, this function should be analogous to the following haskell code:

```haskell
last :: list -> nat option
last [] = Nothing
last [h] = Just h
last h:t = last t
```

This code should have type $\mathbb{L} \rightarrow (\mathbb{N} + ())$. Write this code.

3 Synthesize Prior Code

Write a synthesis derivation for the “last” function you wrote for the prior problem.