1 Bidirectional Typechecking

Provide a bidirectional typechecking derivation for \( f : \forall X X \to X, z : \text{nat} \vdash (\lambda y \; z) : (\text{nat} \to \text{nat}) \to \text{nat} \). \( f \leq \text{nat} \).

2 Multiple Annotations

Provide a bidirectional typechecking derivation for \( \vdash ((\lambda x : \forall A A \to A) : \forall A. \bot \to A) : \bot \to \top) \leq \top \).
You should show the full subtyping derivations, though you don’t need to show the derivations for type variable instantiation.

3 Adding Annotations

Add annotations such that you can provide a bidirectional typechecking derivation for \( n : \text{nat} \vdash (\lambda x. \lambda y \; x) \; n \; (\lambda z \; z) \leq \text{nat} \).

4 Nontypecheckable Program

Come up with an expression of type \( \forall A A \to A \) that cannot be bidirectionally typechecked without adding annotations.