Classical Math is cool
But I like intuitionistic/constructive math more.

Classical Math discusses Truth
$\neg \phi$ means $\phi$ is true.
$\phi \lor \neg \phi$
either $\phi$ is true or $\neg \phi$ is true.

Constructive Math discusses Proof
$\neg \phi$ means I can prove $\phi$.
$\phi \lor \neg \phi$ is not generally true.
It is not the case that, for all \( \psi \),
we have a proof of \( \psi \) or a proof of \( \neg \psi \).

See Gödel’s Incompleteness Thm.

Constructive Math is Sound but not Complete

\[ \vdash \psi \implies \models \psi \]

\[ \models \psi \implies \vdash \neg \psi \]

It is complete with respect to a different semantics using Kripke structures.

We will not go into this in further detail.
Constructive Math "includes" Classical math

\[ \Gamma \vdash \phi \Rightarrow \Gamma \vdash \phi \]

\[ \Gamma \vdash \phi \Rightarrow \Gamma \vdash \neg \neg \phi \]

"But how could \( \neg \neg \phi \) not be true??"

"Don't worry, it's not not not true"

\[ \Gamma \vdash \phi \leq 2^{\text{Prop}} \times \text{Prop} \]

The right side only has 1 prop
This is really the big change that..."
The rest of the rules follow the pattern of "introduction" forms and "elimination" forms. These forms will show up again and again, from usage in ATP, type inference, and more.

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<th>Introduction</th>
<th>Elimination</th>
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<td>$\Gamma \vdash \phi \land \psi$</td>
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Note, we've omitted $\ell$. It is much easier, moving forward, to think of $-\ell$ as $\ell \Rightarrow I$. 