Representing subsets of the set of states.

Let $M = (S, \to, L)$ be a CTL model

$L : S \to \mathcal{P}ow(Atoms)$

Assume a fixed ordering on Atoms, say $x_1, x_2, .., x_n$

① Represent each state by the vector $(v_1, .., v_n)$ of truth values of $x_1, x_2, .., x_n$ in that state.

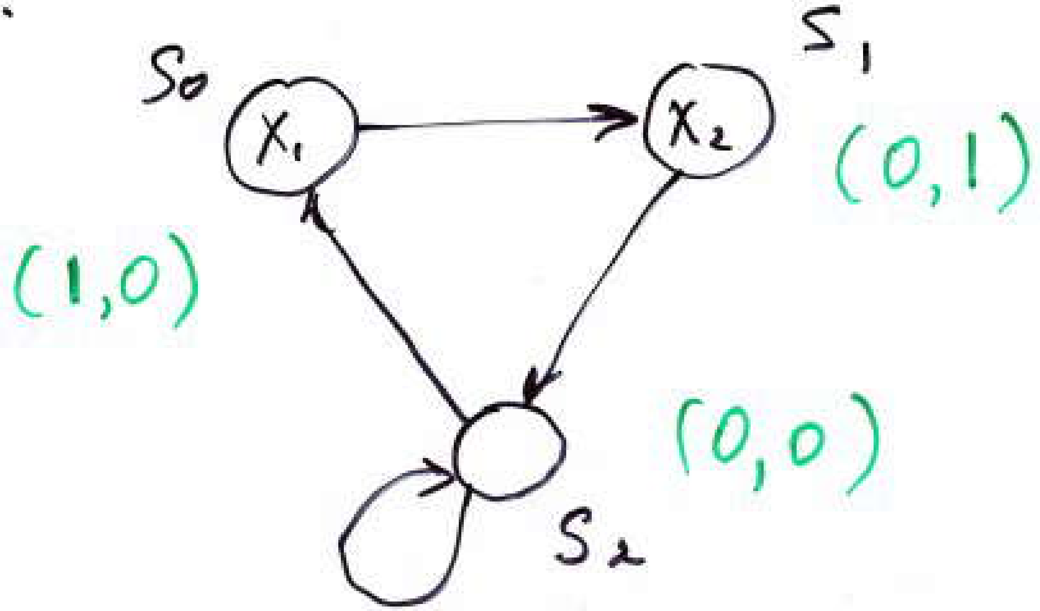
e.g. state labelled $\{x_1, x_3\}$ is represented by $(\underbrace{1, 0, 1, 0 \dots 0}_{n})$

$$v_i \text{ is } \begin{cases} 1 \text{ if } x_i \in L(s) \\ 0 \text{ if } x_i \notin L(s) \end{cases}$$

② Encode vector $(1010\ldots0)$ as boolean function

$$x_1 \cdot \overline{x}_2 \cdot x_3 \cdot \overline{x}_4 \cdot \ldots \cdot \overline{x}_n$$

Ex.



③ Representing subsets of S:

set of states	bool vectors	bool. functions
$\emptyset$	—	0
$\{s_0\}$	$(1,0)$	$x_1 \cdot \overline{x}_2$
$\{s_2\}$	$(0,0)$	$\overline{x}_1 \cdot \overline{x}_2$
$\{s_0, s_2\}$	$(1,0), (0,0)$	$x_1 \cdot \overline{x}_2 + \overline{x}_1 \cdot \overline{x}_2$

How to pick a bool. function better?

Ex. $\{s_0, s_1\}$, bool. vectors $(1,0)$, $(0,1)$

Notice: these truth assignments must satisfy our function, but $(0,0)$ must not (o.w. we'll get $\{s_0, s_1, s_2\}$).

x_1, x_2	f
0 0	0
1 0	1
0 1	1
1 1	1/0 (don't care)

Can pick

$x_1 \cdot \overline{x_2} + \overline{x_1} \cdot x_2$

or, better:

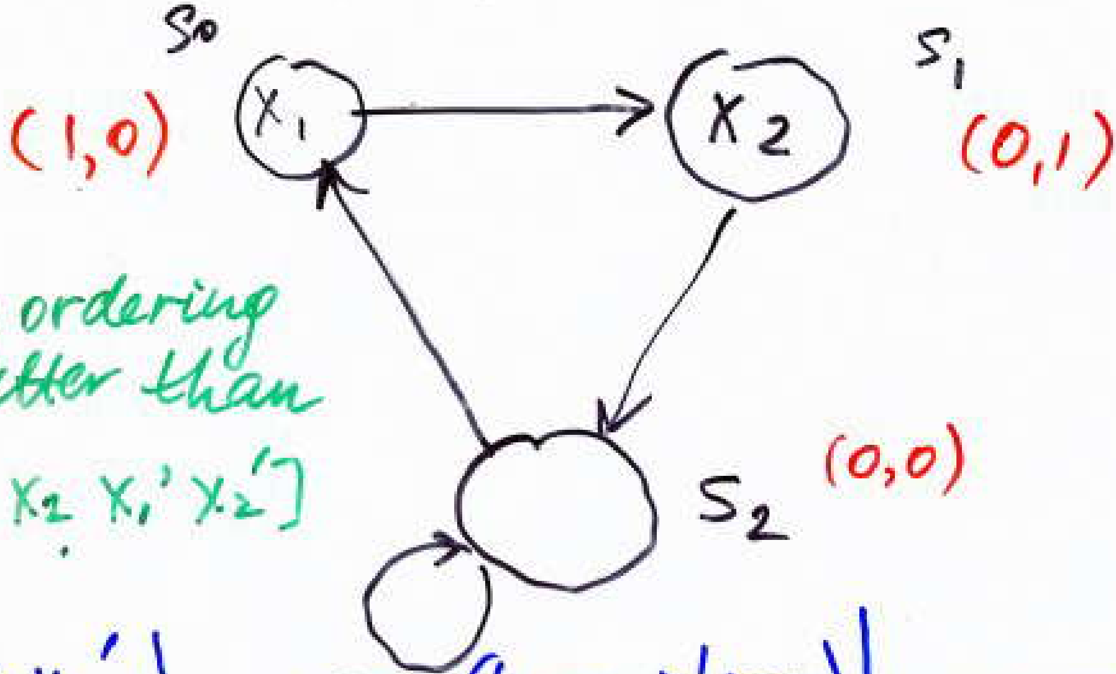
$x_1 + x_2$

Representing the transition relation "$\rightarrow$".

Notice: "$\rightarrow$" is a subset of $S \times S$

Represent each $s \rightarrow s'$ as a pair of bool. vectors:

$$((v_1, v_2, \ldots, v_n), (v_1', v_2', \ldots, v_n'))$$



this ordering is better than $[x_1\, x_2\, x_1'\, x_2']$

$x_1\, x_1'\, x_2\, x_2'$	→ (transitions)	$s \to s'$	f
0 0 0 0	$(\bar{x}_1 \bar{x}_2) \to (\bar{x}_1' \bar{x}_2')$	$s_2 \to s_2$	1
0 0 0 1	$(\bar{x}_1 \bar{x}_2) \to (\bar{x}_1' x_2')$	$s_2 \to s_1$	0
0 0 1 0			0
0 0 1 1			0
0 1 0 0			1
0 1 0 1	$(\bar{x}_1 \bar{x}_2) \to (x_1' x_2')$	$s_2 \to ?$	–
0 1 1 0			0
0 1 1 1			–
1 0 0 0			0
1 0 0 1			1
1 0 1 0			–
1 0 1 1			–
1 1 0 0			0
1 1 0 1			–
1 1 1 0			–
1 1 1 1			–

$$\vec{f} = \bar{x}_1 \cdot \bar{x}_2 \cdot \bar{x}_1' \cdot \bar{x}_2' + \bar{x}_1 \cdot \bar{x}_2 \cdot x_1' \cdot \bar{x}_2' + \dots$$

lines with 1

Building OBDDs

Replace each "don't care" lines with 0 or 1, whichever reduces the number of variables:

e.g.

x_1	x_1'	x_2	x_2'	
0	1	0	0	1
0	1	0	1	~~X~~ 1

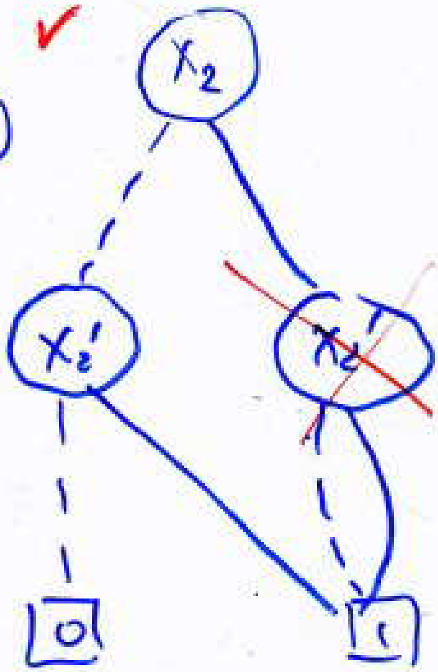
this choice eliminates x_2'!

Picking possible values for (*) :

x_1	x_1'	x_2	x_2'	1	2	3	4
1	0	0	0	0	0	0	0
1	0	0	1	1	1	1	
1	0	1	0	1	0	0	1
1	0	1	1	1	1	0	0

these values do not affect our choices

1



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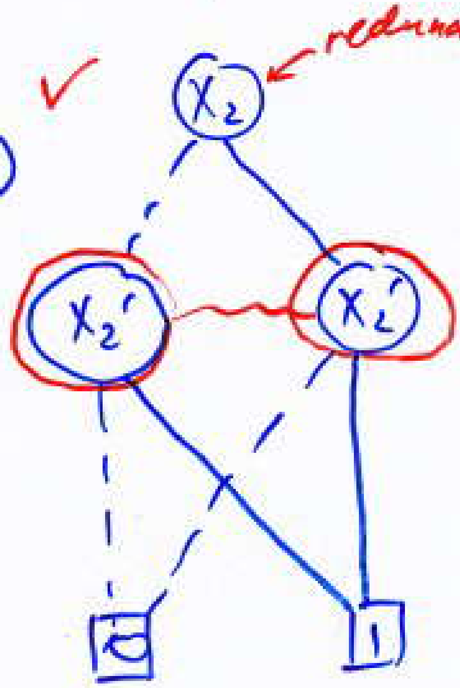

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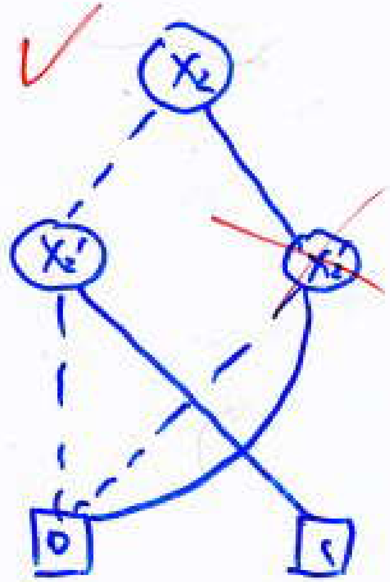

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