

## Tseitin's Polytime Transformation to CNF.

Assume our formula is in NNF (negation normal form), where all negations are pushed inwards using De Morgan Laws & double negation law.

For every sub-formula  $B$  of  $A$ , let  $P_B$  denote  $B$  if  $B$  is a literal and a new atom if  $B$  is not a literal.

$CNF(A)$  includes:

the "top" clause ( $P_A$ )

and, for each sub-formula

$B$  of  $A$ :

$$(A \rightarrow B) \equiv (\neg A \vee B)$$

1. If  $B$  is  $(C \vee D)$ , include

the clauses for  $(P_B \leftrightarrow (P_C \vee P_D))$

$$(\neg P_B \vee P_C \vee P_D), (\neg P_C \vee P_B), (\neg P_D \vee P_B)$$

2. If  $B$  is  $(C \wedge D)$ , include

the clauses for  $(P_B \leftrightarrow (P_C \wedge P_D))$

$$(\neg P_B \vee P_C), (\neg P_B \vee P_D), (\neg P_C \vee \neg P_D \vee P_B)$$

Th Every t.a. that sat.  $CNF(A)$ ,

sat.  $A$ ,

Every t.a. that sat  $A$  can be extended  
to the one that sat.  $CNF(A)$

$$\text{Ex. } A := ((\underbrace{Q \wedge R}_{X}) \wedge (\underbrace{Q \wedge S}_{Y}))$$

$\text{CNF}(A) :$

$$(\underbrace{P_A}_{(P_A \leftrightarrow P_x \wedge P_y)} \wedge$$

$$(\underbrace{P_x \leftrightarrow (P_Q \wedge P_R)}_{(P_x \leftrightarrow (P_Q \wedge P_R)) \wedge (P_y \leftrightarrow (P_Q \wedge P_S))} \wedge \underbrace{(P_y \leftrightarrow (P_Q \wedge P_S))}_{(P_y \leftrightarrow (P_Q \wedge P_S))})$$

rewrite in clausal form

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$\text{CNF}(A) :$

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$$(\underbrace{P_x \leftrightarrow (P_Q \wedge P_R)}_{(P_x \leftrightarrow (P_Q \wedge P_R)) \wedge (P_y \leftrightarrow (P_Q \wedge P_S))} \wedge \underbrace{(P_y \leftrightarrow (P_Q \wedge P_S))}_{(P_y \leftrightarrow (P_Q \wedge P_S))})$$

rewrite in clausal form