Graph Coloring as SAT

Task:

Given: graph $G = \langle V, E \rangle$

k colours $\{1, \dots, k\}$

Task: Find a proper k-colouring of G

i.e., a function $col: V \to \{1, .., k\}$

such that $(u, v) \in E \Rightarrow col(u) \neq col(v)$

SAT encoding:

Atoms:

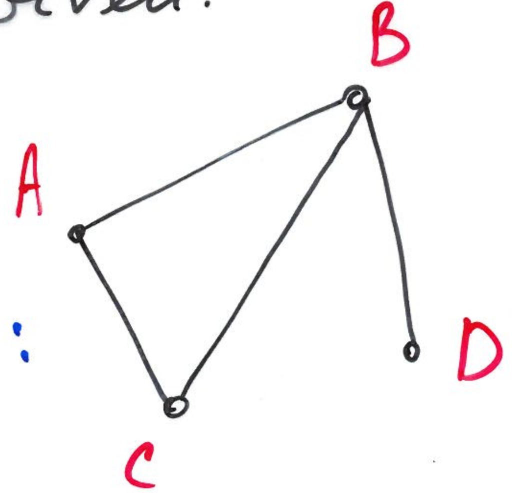
$C_{v,c}$ - meaning $Col(v) = c$

for each $v \in V$ and $c \in \{1, \dots, k\}$

Given:

colours $\{1, 2, 3\}$

G :



$k = 3$

Construct a formula that is satisfiable iff G has a proper k-colouring

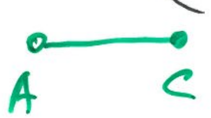
$$\left(C_{A,1} \vee C_{A,2} \vee C_{A,3} \right)$$

vertex A gets some colour

$$\wedge \left(C_{B,1} \vee C_{B,2} \vee C_{B,3} \right)$$

$\wedge$ for C and D

$$\wedge \left(\overline{C_{A,1}} \vee \overline{C_{C,1}} \right) \wedge \left(\overline{C_{A,2}} \vee \overline{C_{C,2}} \right) \wedge \left(C_{A,3} \vee \overline{C_{C,3}} \right)$$



two connected vertices must not be coloured the same colour

$\wedge$ for all other edges

In general

$$\bigwedge_{v \in V} \left(\bigvee_{c \in \{1, \dots, k\}} C_{v,c} \right)$$

$$\wedge$$

$$\bigwedge_{(u,v) \in E} \bigwedge_{c \in \{1, \dots, k\}} \left(\overline{C_{u,c}} \vee \overline{C_{v,c}} \right)$$

$$= \Gamma(G, k)$$

Formula $\Gamma(G,k)$ is satisfiable iff G has a proper k-colouring.

~~and~~ each t.a. gives us a proper colouring