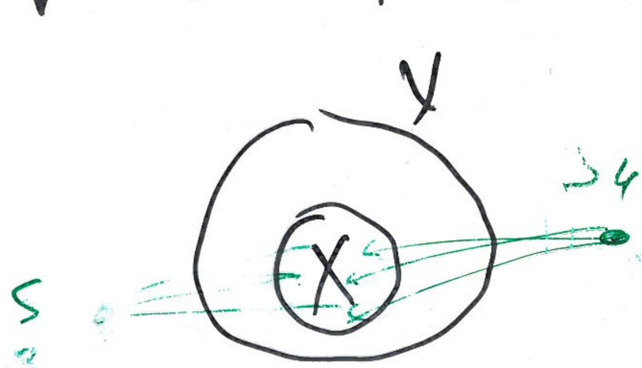
Using the function $F(X) = [\![\varphi]\!] \cup pre_{\forall}(X)$ prove that $[\![AF\varphi]\!]$ is the least fixed point of F. Hence argue that procedure SAT_{AF} is correct and terminates.

Solution:

1) Show F is monotone

Take any two subsets X and Y of S such that $X \subseteq Y$. Show $F(X) \subseteq F(Y)$, that is $[\![\varphi]\!] \cup pre_{\forall}(X) \subseteq [\![\varphi]\!] \cup pre_{\forall}(Y)$ that is $pre_{\forall}(X) \subseteq pre_{\forall}(Y)$.



Take any $s \in pre_{\forall}(X)$.

for all transitions starting from s, $s \to s'$, $s' \in X$.

Since $X \subseteq Y$, $s' \in Y$.

By def. of $pre_{\forall}$, $s \in pre_{\forall}(Y)$

So $pre_{\forall}(X) \subseteq pre_{\forall}(Y)$ ∎

How to show monotonicity formally