2) Show that by iterating the operator we'll obtain $[\![AF\varphi]\!]$

F is monotone. By th*, $F^{n+1}(\emptyset)$ is the least fix point of F.

Therefore, it suffises to show that this set equals $[\![AF\varphi]\!]$.

Observe what sets of states we obtain by iterating F:

$$F^1(\emptyset) = [\![\varphi]\!] \cup pre_{\forall}(\emptyset)$$

$= [\![\varphi]\!]$, all states where φ holds. Choose $i=0$ (according to the def. of AF, need to find s_i where φ holds in all paths)

We have:

$$F^2(\emptyset) = [\![\varphi]\!] \cup \mathrm{pre}_\forall(F^1(\emptyset))$$

It tells us that the elements of $F^2(\emptyset)$ are all those $s_0 \in [\![AF\varphi]\!]$ where we choose $i \leq 1$. to witness F

By math. induction, ~~w~~

$F^{k+1}(\emptyset)$ is the set of all states s_0 for which we choose $i \leq k$ to secure $s_0 \in [\![AF\varphi]\!]$.

Since this holds for all k, we see that the union of $F^{k+1}(\emptyset)$ $k \geq 0$ is just $[\![AF\varphi]\!]$.

Since $F^{n+1}(\emptyset)$ is a fixpoint of F, we see that that union is just $F^{n+1}(\emptyset)$.

Thus, we proved that $[\![AF\varphi]\!]$ is the least fixpoint of F.

Since F is the operator used in SAT_{AF}, this algorithm terminates and is correct.